Stability Analysis and Control for 3D Biped Robot HYDROïD

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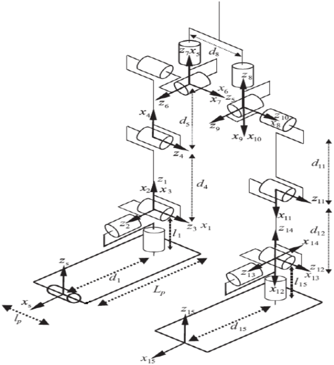
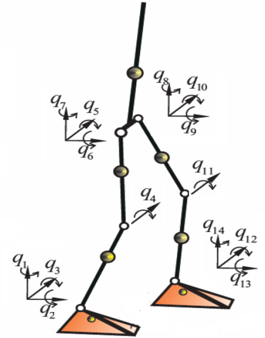
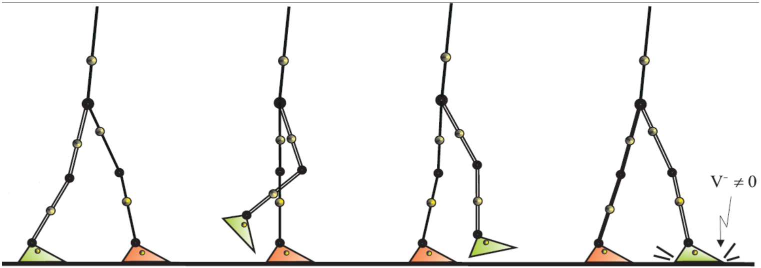
The studied robot HYDROÏD





- structure : torso + $2\times$(hip $3D$ + knee $1D$ + ankle $3D$) + flat-feet
- configuration vector : $q = [q_1, q_2, \ldots, q_{14}]'$
- Torque vector : $\Gamma = [\Gamma_1, \Gamma_2, \ldots, \Gamma_{14}]'$

Complete walking phase



- double support : instantaneous
- simple support : flat-foot
- transition : impact
- impact model considering exchange of the leg :

$$\begin{cases} q^+ = Eq^- \\ \dot{q}^+ = E(I(q^-)\dot{q}^-) \end{cases} \tag{1}$$

Dynamic model during the single support phase

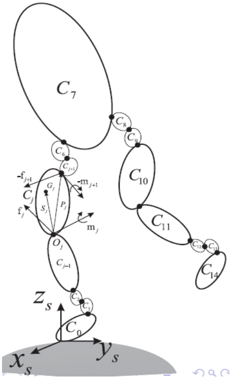
The Newton-Euler algorithm is used to calculated the dynamic model :

$$\omega_0 = 0, \dot{\omega}_0 = 0, V_0 = 0, \dot{V}_0 = -g$$

$$\begin{bmatrix} \mathbf{F} \\ \mathbf{M} \\ \Gamma \end{bmatrix} = \mathbf{NE}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$$

avec :

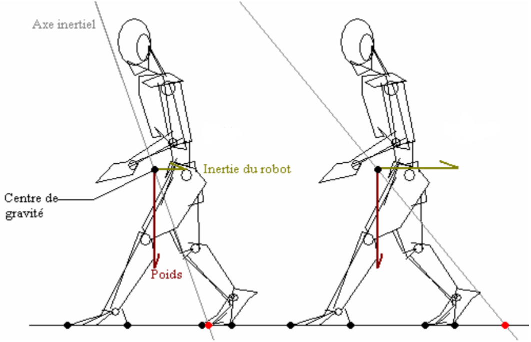
- $\Gamma \in \mathbb{R}^{14}$: torques of actuated joints
- $\mathbf{M} \in \mathbb{R}^3$ and $\mathbf{F} \in \mathbb{R}^3$: moment and force vector exerted by the ground on the stance foot.
- $\dot{\mathbf{q}} \in \mathbb{R}^{14}$: joint velocity
- $\ddot{\mathbf{q}} \in \mathbb{R}^{14}$: joint acceleration



Equilibrium dynamics during the single support phase

Zero Moment Point :

- ZMP is the point on the ground at which the net moment of the inertial forces and the gravity forces has no component along the horizontal axes.

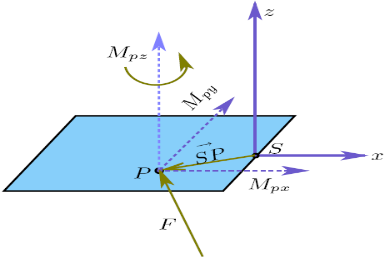


- Dynamically balanced gait : $ZMP \equiv CoP$ (Centre of Pressure)

Equilibrium dynamics

- $\mathbf{M_P} = \mathbf{M_S} + \overrightarrow{PS} \wedge \mathbf{F_S}$
- The force and moment exerted by the ground on the stance foot :

$$\begin{bmatrix} \mathbf{F_S} \\ \mathbf{M_S} \end{bmatrix} = \mathbf{NE}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$$

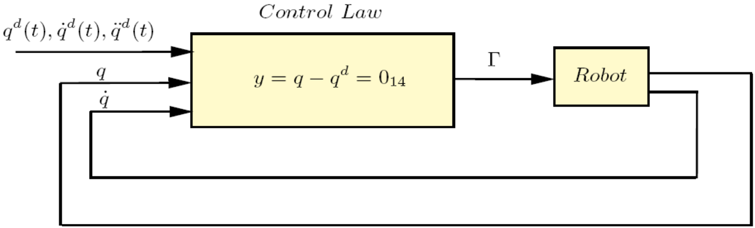


- The position of ZMP in R_s : $\overrightarrow{SP} = (l_x, l_y, 0)$
-

$$\begin{bmatrix} \mathbf{M_{Px}} \\ \mathbf{M_{Py}} \end{bmatrix} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, l_x, ly) = \color{red}{W}\ddot{\mathbf{q}} + \color{red}{H} \tag{2}$$

Control Principle

General control law :



Problem :

- It is difficult to satisfy the constraint of ZMP.

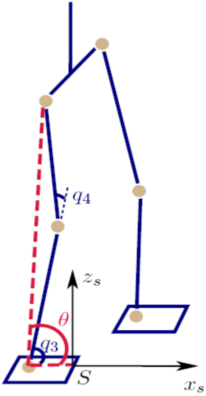
Time-invariant Reference Trajectory

$$q^d(t) \Rightarrow q^d(\theta)$$

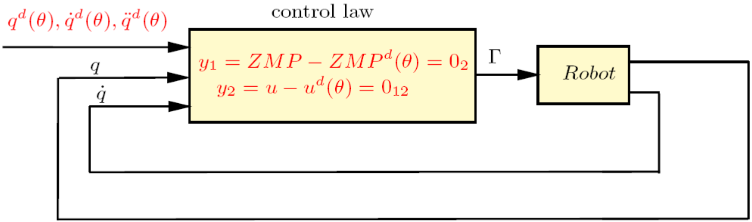
- The time t is replaced with a quantity θ.
- It is successfully used for 2D Rabbit.

Advantages :

- It requires a path in the joint space.
- It doesn't need the temporal evolution.
- It permits to satisfy the constraint of ZMP.



Principle of our control



- The ZMP is controlled. (2 equations)
- 12 controlled outputs can be chosen. (12 equations)
- 2 configurations of the robot are uncontrolled.

Reference Trajectory

- The time t is replaced with a quantity θ that is strictly monotonic along a typical walking gait :

$$\theta = q_3 + 0.5 q_4$$

- The desired configurations of the robot $\mathbf{q^d}$:

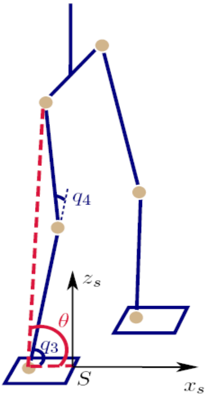
$$\mathbf{q^d} = Bezier(s)$$

avec :

$$s = \frac{\theta - \theta_i}{\theta_f - \theta_i}$$

$$s = 0 \Leftrightarrow t = 0,$$

$$s = 1 \Leftrightarrow t = T$$



The control outputs

$$\begin{cases} y_1 = ZMP - ZMP^d(\theta) \\ y_2 = u - u^d(\theta) \end{cases} \tag{3}$$

- For a 3D robot with 14 actuated joints, when the 2 positions of the ZMP is controlled only other 12 outputs can be chosen.

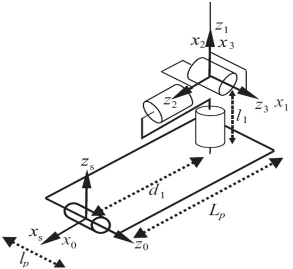
$$\mathbf{u} = M\mathbf{q} = M_1\,\mathbf{v} + \mathbf{Q} \tag{4}$$

where :

$$\mathbf{v} = [q_2, \theta]'_{2\times 1}$$

$$\theta = q_3 + 0.5q_4$$

$$\mathbf{Q} = [q_1, q_4, \ldots, q_{14}]'_{12\times 1}$$



$M_1(12 \times 2)$ influence the walking stability of the 3D robot.

Control Law

- In order to let $y_1 = ZMP - ZMP^d(\theta) = 0$, W, H can be obtained to satisfy :

$$\begin{bmatrix} \mathbf{M_{Px}} \\ \mathbf{M_{Py}} \end{bmatrix} = W\ddot{\mathbf{q}} + H = 0_2 \tag{5}$$

- In order to let $y_2 = u - u^d(\theta) = 0$, the PD controller is used :

$$\ddot{\mathbf{u}}_{12} = \ddot{\mathbf{u}}^d(\theta) - K_d\dot{y}_2 - K_p y_2$$

- 14 equations are used to solve $\ddot{q}(14 \times 1)$.
- The torques are calculated with Newton-Euler algorithm :

$$\Gamma = \mathbf{NE}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \tag{6}$$

Example of $M_1 = zeros(12, 2)$

- The controlled outputs :

$$\mathbf{u} = M_1\mathbf{v} + Q \tag{7}$$

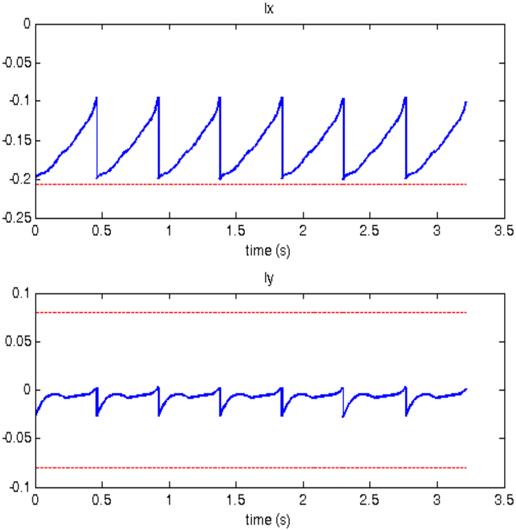
with :

$$\mathbf{Q} = [q_1, q_4, \ldots, q_{14}]'_{12\times 1}$$

- $$y_2 = [q_1, q_4, \ldots, q_{14}]'_{12\times 1} - \left[{q_1}^d, {q_4}^d, \ldots, {q_{14}}^d\right]'_{12\times 1}$$

The walking gait obtained with simple model

Positions of ZMP with $M_1 = 0_{12}$



Zero dynamics

The zero dynamics was defined to describe the behavior of the system when the outputs are assumed to be 0.

- If $u = u^d$:

$$\mathbf{u} = M_1\mathbf{v} + Q = M_1\mathbf{v^d} + Q^d \tag{8}$$

  there exists :

$$Q = Q^d - M_1(v - v^d) \tag{9}$$

- The configurations variables q is rewritten with the unactuated variables v :

$$q = q^d + \tau^{-1}[1 \quad -M_1]^T(v - v^d) \tag{10}$$

The reduced dimension system for stability analysis

$ZMP = ZMP^d$

$\ddot{u} = \ddot{u}^d - K_d(\dot{u} - \dot{u}^d) - K_p(u - u^d)$

$$\boxed{\begin{array}{l} W\ddot{\mathbf{q}} + H = 0_2 \\ \text{28 dimensions} \end{array}} \xrightarrow{u = u^d} \boxed{\begin{array}{c} W_{hzd}(V, \dot{V}, M_1)\ddot{V} + H_{hzd}(V, \dot{V}, M_1) = 0_2 \\ \text{4 dimensions} \end{array}}$$

- Considering $v = [q_2, \theta]$, and $\dot{q}_2$ depends of θ and $\dot{\theta}$, the stability of the closed loop system can be analyzed in 3 dimension space.
- The properties of zero dynamics is influenced by M_1.

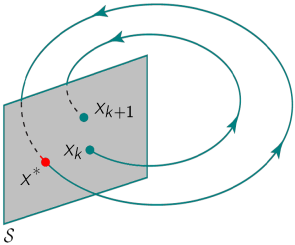
Application of Poincaré method in reduced dimension

- Defining :

$$x = [q_2, \dot{q}_2, \dot{\theta}]'$$

$$x^* = [q_2(s^*), \dot{q}_2(s^*), \dot{\theta}(s^*)]'$$

$$s^* = 0.6$$



- Defining :

$$\delta x_k = x_k - x^*$$

The poincaré section is linearized as :

$$\delta x_{k+1} = A\delta x_k$$

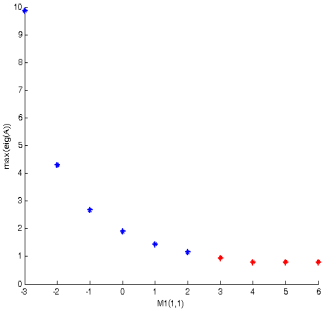
$max\,|eig(A)| < 1 \Leftrightarrow$ the system is stable.

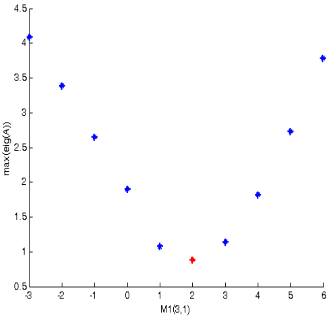
Examples of stability analysis with different $M_{1(12\times 2)}$

- The controlled outputs :

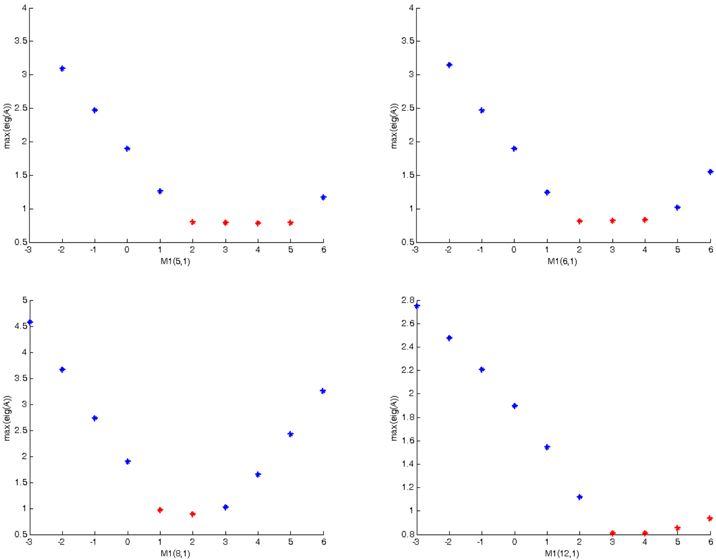
$$\mathbf{u} = M\mathbf{q} = M_1\,\mathbf{v} + \mathbf{Q} \qquad (11)$$

- $max\,|eig(A^z)|$ of $M_1(j,1), j = 1,\ldots,12$ are studied when the other 23 components of M_1 are zero.

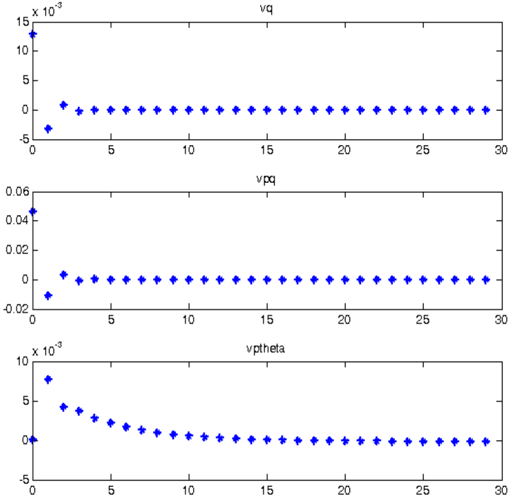




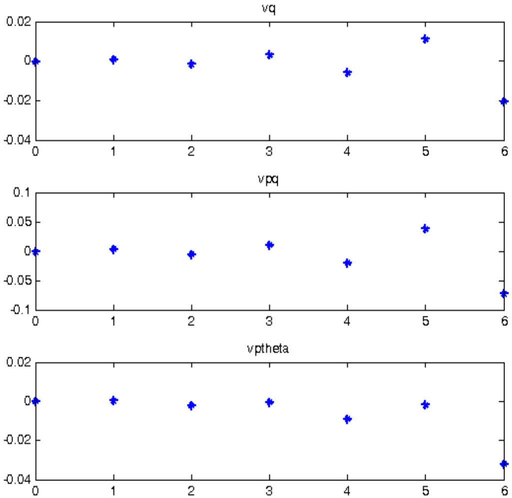
$max\,|eig(A^z)|$ versus $M_1(j,1)$, j = 5, 6, 8, 12



Tracking errors of unactuated variables $q_2, \dot{q}_2, \dot{\theta}$ at the end of every step with $M_1(5,1) = 3$, $max\,|eig(A^z)| = 0.7899$



Tracking errors of unactuated variables $q_2, \dot{q}_2, \dot{\theta}$ at the end of every step with $M_1 = 0_{12}$, $max\,|eig(A^z)| = 1.8972$



The walking gait obtained with simulator of robot

Conclusions and Perspectives

Conclusions :

- A new control law based on ZMP is designed for 3D biped robot.
- Stability is researched with application of the Poincaré map.

Perspectives :

- Compare with the classical control method.
- Consider the rotation of the foot.
- Control the direction of the trajectory.
- Experiments.

Thanks for your attention !